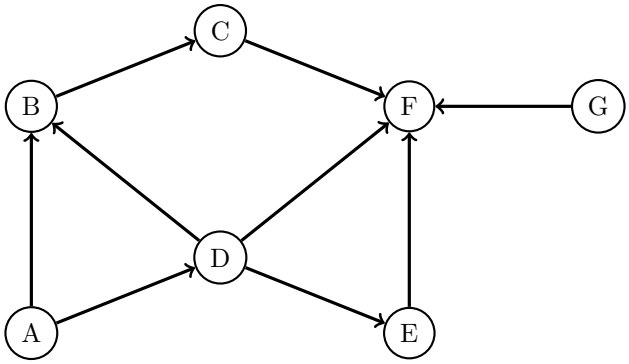
CS 61B Spring 2019

Graphs

Discussion 11: April 10, 2019

1 Graph Representations



1.1 Write the graph above as an adjacency matrix, then as an adjacency list. What would be different if the graph were undirected instead?

2 Topological Sorting

2.1 Give a valid topological sort of the graph above. For your reference, some orderings of the graph are provided below.

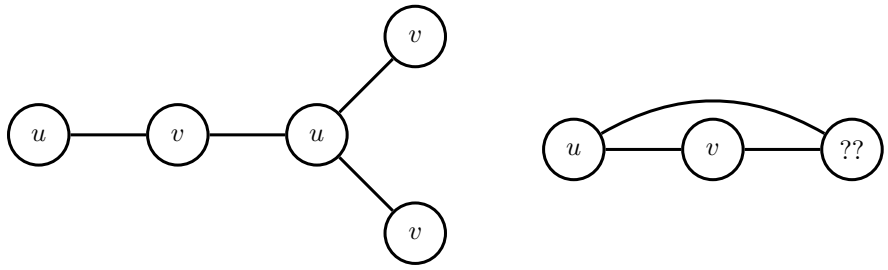
```
DFS preorder: ABCFDE (G)
DFS postorder: FCBEDA (G)
BFS: ABDCEF (G)
```

2.2 There are two requirements that a graph must satisfy in order for there to be a valid topological sorting of the graph. What are they?

2.3 *Extra:* Why does the method to compute a topological sort work?

3 Graph Algorithm Design

3.1 An undirected graph is said to be bipartite if all of its vertices can be divided into two disjoint sets U and V such that every edge connects an item in U to an item in V. For example below, the graph on the left is bipartite, whereas on the graph on the right is not. Provide an algorithm which determines whether or not a graph is bipartite. What is the runtime of your algorithm?



3.2 Consider the following implementation of DFS, which contains a crucial error:

```
create the fringe, which is an empty Stack
push the start vertex onto the fringe and mark it
while the fringe is not empty:
    pop a vertex off the fringe and visit it
    for each neighbor of the vertex:
        if neighbor not marked:
            push neighbor onto the fringe
            mark neighbor
```

Give an example of a graph where this algorithm may not traverse in DFS order.

3.3 *Extra:* Provide an algorithm that finds the shortest cycle (in terms of the number of edges used) in a directed graph in $O(EV)$ time and $O(E)$ space, assuming $E > V$.